Andronov bifurcation and excitability in semiconductor lasers with optical feedback

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We experimentally investigate the dynamical behavior of a semiconductor laser with optical feedback. We show that noise plays an important role close to the instability threshold, while determinism controls the so-called coherence collapse regime. We identify the bifurcation which is at the origin of the low frequency fluctuations. It is the result of a collision between a stable fixed point and a saddle point (Andronov's bifurcation). We provide experimental proof that the laser with optical feedback behaves as an excitable medium. [S1063-651X(97)00805-2]

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I. INTRODUCTION

The dynamical behavior of semiconductor lasers with external optical feedback was extensively studied [1]. Previous experiments showed that the laser intensity is unstable with aperiodic oscillations in a large range of parameter values [2–4]. Three main regimes of operation were observed: (I) intensity constant in time, (II) low frequency fluctuations (LFF), and (III) coherence collapse (CC) [5]. It is almost already accepted that the fluctuations have a deterministic origin rather than being produced by noise. However, the dynamical process yielding LFF and CC is not yet fully understood.

From the theoretical point of view, the system can be modeled using Maxwell-Bloch equations. Usually the atomic polarization can be adiabatically eliminated due to its fast relaxation rate, and the two relevant variables are the electromagnetic field and the carrier density (or population inversion) [6]. The so-called Lang-Kobayashi equations [7] are obtained assuming a weak feedback and single mode operation of a unidirectional ring cavity laser. On the other hand, the linewidth enhancement factor (α) plays the role of a detuning between atomic and cavity resonances. Analytical studies of such equations gave multiple steady-state solutions. Linear stability analysis showed the existence of at least one stable stationary solution in a large range of all parameter values, and the existence of pairs of solutions called modes and antimodes [8,9]. The modes are generally unstable due to a Hopf bifurcation and the antimodes are saddle points. Several attempts have been made in order to explain why the system tends to oscillate rather than to remain at the stable fixed point with intensity constant in time [9]. However, none of them are based on basic principles and they use assumptions not already confirmed experimentally. Numerical simulations show qualitative agreement with some but not all experimental results.

Here we use a different approach in order to understand the dynamical origin of the observed phenomena. We do not make a direct comparison between a model and the experiment, but we perform a series of measurements to characterize the intensity fluctuations of the laser. This is used to identify the possible bifurcations capable of generating such a dynamical behavior.

II. DYNAMICAL ORIGIN OF THE REGIMES

The experimental setup is shown in Fig. 1, and it is similar to the one described in Ref. [5]. The laser is a Hitachi HLP 1400. The temperature of the laser is stabilized to better than 0.01 °C. An AR coated collimator is placed at the laser output in order to reduce the beam divergence. An external mirror of 95% reflectivity is placed at a distance from the laser output. This distance can be varied between 0.1 and 0.5 m. The intensity output is detected by 5 GHz bandwidth silicon avalanche photodiodes. The signal is analyzed by a spectrum analyzer (22 GHz bandwidth), and a 7200A LeCroy digital oscilloscope (500 MHz analog bandwidth). Part of the output beam is reflected into a scanning Fabry-Pérot analyzer to check the spectral characteristics of the laser output.

The laser without optical feedback operates in a single longitudinal mode for currents between 67 and 100 mA. Near threshold the spectrum shows multimode operation or instabilities associated with mode jumping.

The amount of feedback depends both on the tilt of the mirror and the collimation of the beam. The output intensity at different values of the pumping current becomes unstable depending on the amount of feedback. In fact, for a given

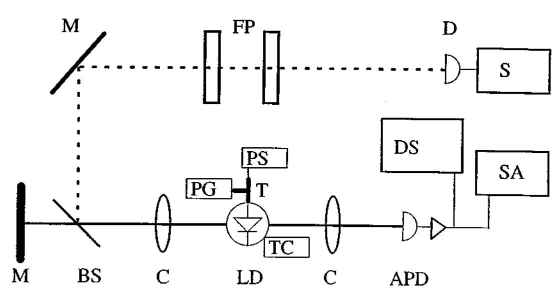

FIG. 1. Experimental setup: LD, laser diode; TC, temperature controller; APD, avalanche photodiode; SA, spectrum analyzer; C, collimators; D, detector; FP, scanning Fabry-Pérot; BS, 50/50 beam splitter; M, mirror; PG, pulse generator; T, bias tee; DS, digital scope; S, scope; and PS, power supply.

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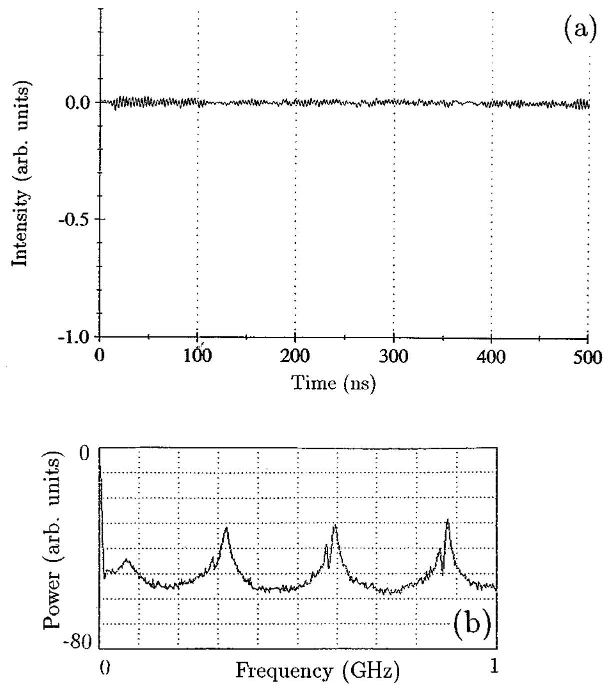

FIG. 2. Region I (see text): (a) Temporal behavior of the intensity signal, (b) power spectrum of (a).

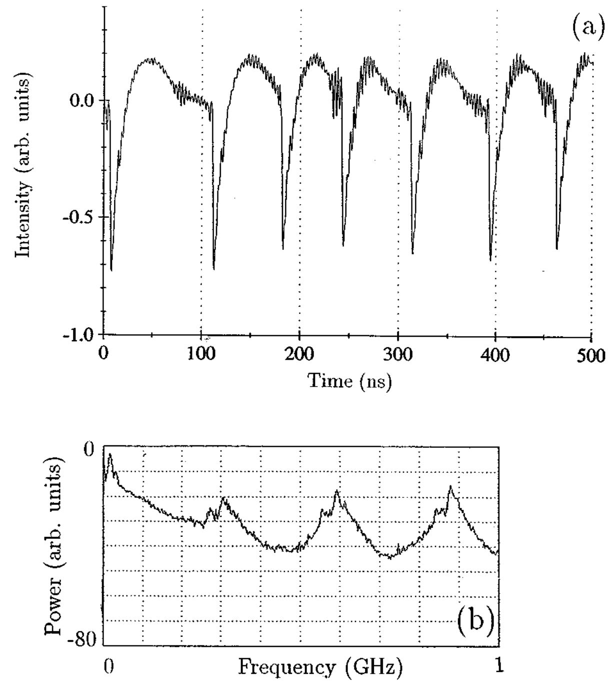

FIG. 3. Same as in Fig. 2 for region II (see text).

position of the collimator the intensity is unstable in the whole range of current. We choose the position of the collimator for which the reduction of the threshold current is maximized.

Analyzing the intensity as a function of time we identify the three different regimes of operation already described in Ref. [5].

For pump values smaller than or of the order of the threshold for the solitary laser (regime I), the intensity is almost constant in time [Fig. 2(a)]. A small oscillation can be resolved in the power spectrum [Fig. 2(b)], which shows a peak corresponding to the external cavity mode separation. The optical spectrum shows that the laser works on a single solitary laser mode. For pump values exceeding the threshold of the solitary laser (regime II), the intensity as a function of time shows trains of pulses separated by regions of constant intensity [Fig. 3(a)]. The power spectrum grows in the low frequency region [Fig. 3(b)] and the optical spectrum shows that the laser works on different solitary laser modes. This behavior is called low frequency fluctuations. At higher pumping rates (regime III), the frequency of the pulsations of the intensity increases [Fig. 4(a)]. The power spectrum shows the appearance of a peak over a broadband signal [Fig. 4(b)]. The system entered the region of the so-called coherence collapse. The three regimes give an overall description of the behavior of the system. However, a more detailed analysis is required in order to understand the mechanism underlying the dynamics. We analyze below the LFF and CC regimes using statistical and averaging methods and special measurements like return maps.

The first relevant experimental test consists in performing an averaging of the time evolution between consecutive pulses overcoming a prefixed intensity threshold. We trigger the oscilloscope on the maximum of the derivative of the intensity each time the intensity becomes lower than the threshold value, and superpose the traces of 10^4 events. In Fig. 5(a) we observe that the fast oscillations between consecutive pulses are washed out by the averaging process in the LFF regime, except for two or three periods just before the giant pulse. In the coherence collapse instead [Fig. 5(b)] the structure of the oscillations remains.

The above result suggests that deterministic terms play a relevant role in the CC regime. To provide more insight into the behavior of the system in the LFF regime, we analyzed the statistics of the time interval between pulses. An impor-

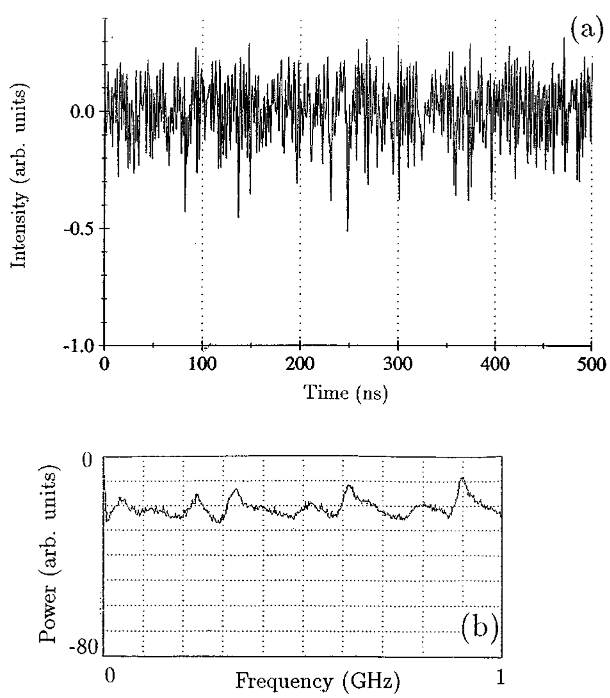

FIG. 4. Same as in Fig. 2 for region III (see text).

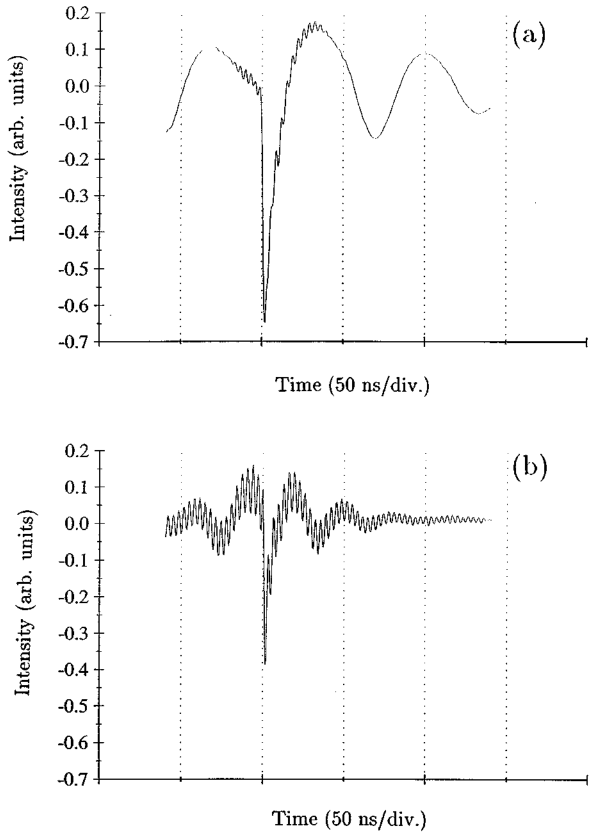

FIG. 5. Average of the temporal behavior over 10^4 successive pulses: (a) regime II, (b) regime III.

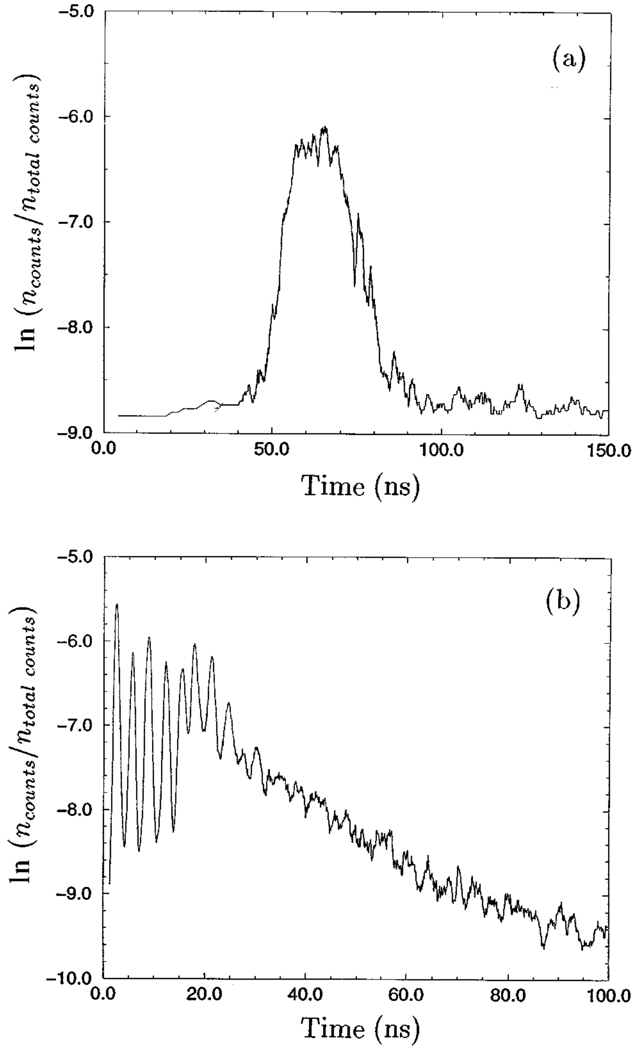

FIG. 6. Histogram of time intervals T: (a) regime II, (b) regime III.

tant indication is given by the distribution of times T in the two regimes. In the LFF regime the histogram of T decreases exponentially beginning at a finite T value [Fig. 6(a)]. On the other hand, in the CC regime, there is a structure formed by discrete peaks indicating that certain time intervals are preferred by the system [Fig. 6(b)].

In Fig. 7 we show the return maps of the signal obtained plotting the time T between successive dropouts of the intensity. This diagram is made by digitizing the temporal signal and memorizing the times θ_n for which the intensity of the nth pulse reaches a prefixed given value I_0, and evaluating $T_n = \theta_n - \theta_{n-1}$. We have chosen the threshold I_0 in a region of the intensity for which the maps are independent from that value. This return map is equivalent to performing a Poincaré section of the system at constant intensity. In fact, if the signal is periodic, it gives a single point in the return map, or a finite set of discrete points. If it is aperiodic, the different points are distributed over an extended region. Thus at least the presence of well defined structures in the return maps indicates the deterministic origin of the dynamics. We perform the measurements over samples of more than 10^4 pulses.

Figure 7(a) is obtained when the pump rate is close to the transition between regions I and II. The return map shows a cloud of points without structure as we can expect from a system where the noise controls its dynamics. As a further test, we plotted also the k-order return maps (T_n vs T_{n-k}) for several k values. Such analysis has not revealed the presence of higher-order correlations. Increasing the pump, the coherence collapse regime is reached, and the return map is a collection of points aligned into straight perpendicular and horizontal lines [Fig. 7(b)].

Therefore we can conclude the following. (1) In the CC regime, deterministic terms play the relevant role in describing the dynamical behavior of the system giving phase correlation among the interval T. (2) In the LFF regime the system loses memory during the interval T, suggesting that the noise acts as a trigger for the high amplitude pulses.

The histograms allow us to measure also the average time ($\langle T \rangle$) between pulses as a function of a normalized control parameter, $e = (J - J_{\rm th})/J_{\rm th}$, where J is the pump current and $J_{\rm th}$ is the current at laser threshold. Our measurement confirms the results obtained in Ref. [10]. However, it is worthwhile to note that the standard deviation of $\langle T \rangle$ is almost 100% while the standard deviation over the amplitude and width of the pulses is only 10% in the LFF regime, confirming the deterministic origin of the dynamical evolution of the pulse itself (‘‘coherent’’ buildup of the pulses).

III. EXCITABILITY

From the previous measurements, it follows that there must exist at some parameter value a bifurcation point which

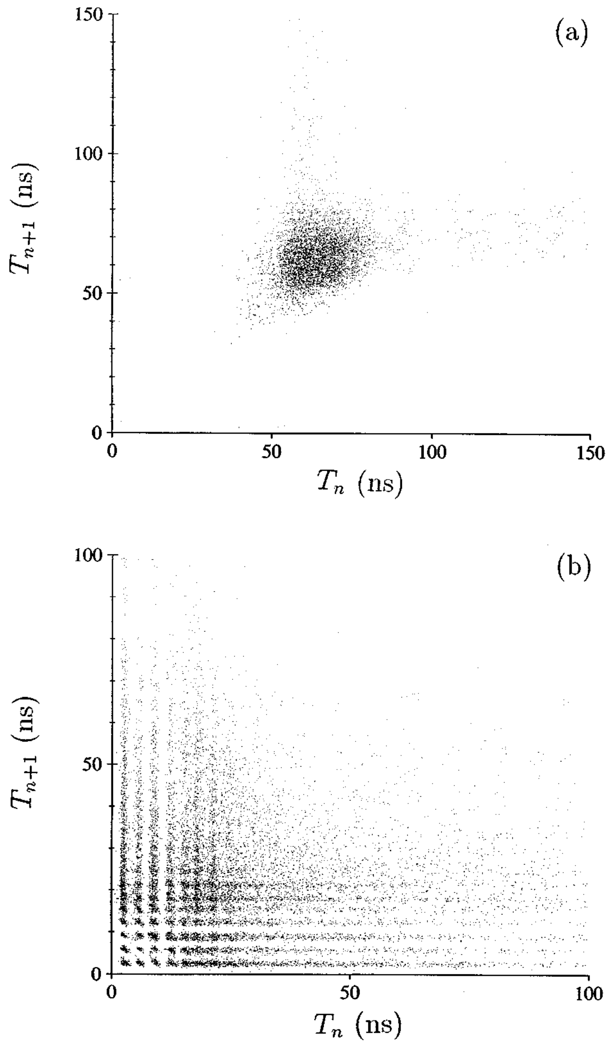

FIG. 7. Return maps of the time intervals T between successive pulses in regime II (a), and in regime III (b).

is anticipated by the presence of noise. The time T is then very sensitive to the noise amplitude confirming the previous hypotheses [10,11] which describe the process as intermittency.

In nonlinear dynamics two possible bifurcations can lead to an oscillatory behavior: a Hopf bifurcation or an Andronov bifurcation [12].

Considering the observations presented above, we immediately discard the existence of a supercritical Hopf bifurcation in our system because it would lead to small amplitude pulsing with a well defined frequency.

In our case the passage between regime I and regime II is characterized by the existence of high amplitude pulses. Then, from the dynamical point of view, such behavior may correspond to a subcritical Hopf or an Andronov type bifurcation. However, the subcritical Hopf bifurcation is usually detected by the presence of generalized multistability between two different possible states of the system that we never observed in our experiment. An Andronov bifurcation is instead characterized by an arbitrary amplitude of the oscillations while its frequency tends to zero as we approach the bifurcation point.

Then, the LFF regime is seen as a consequence of two joint effects: the existence of an Andronov bifurcation as we increase the pump parameter, and the noise which drives the system anticipating the bifurcation and avoiding a periodic regime. After the parameter value largely overcomes the bifurcation point, the frequency of the oscillations increases, deterministic processes are less sensitive to the amount of noise, but the nonlinearities are so important that the behavior is chaotic generating the CC regime.

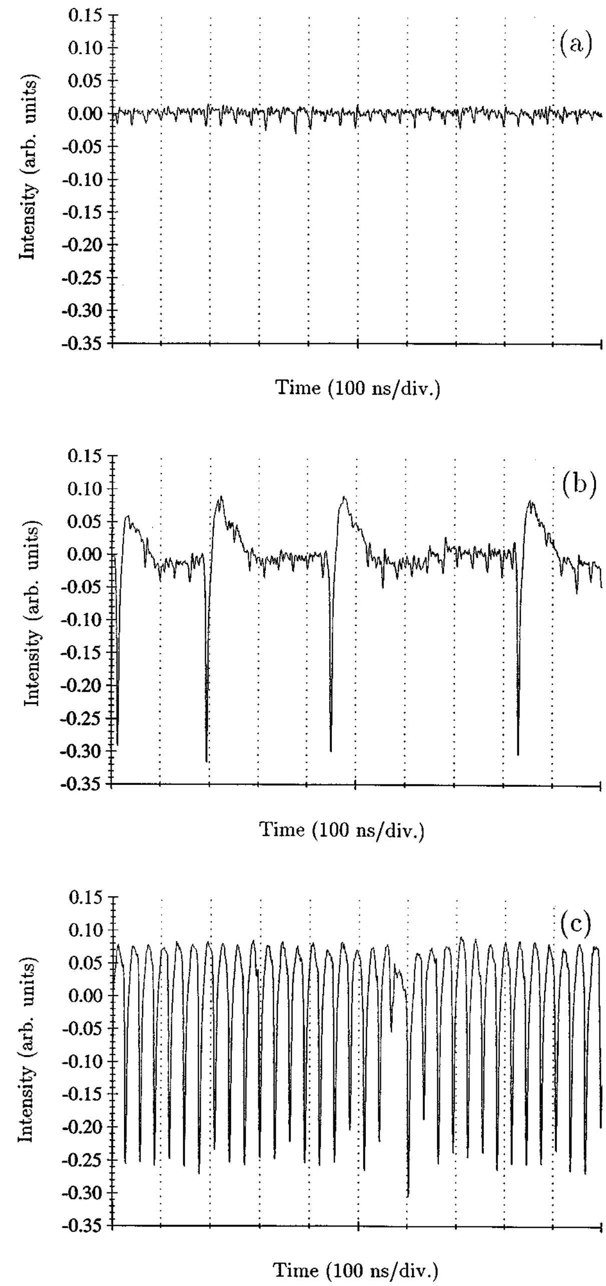

FIG. 8. Intensity of the system when a small amplitude perturbation (width: 60 ps; period: 30 ns) is added to the pumping current; (a) amplitude of the pulse: 2.6 mA; (b) amplitude of the pulse: 3 mA; (c) amplitude of the pulse: 10 mA.

A system possessing an Andronov bifurcation is excitable [12]. An excitable medium can be recognized by its response to fast variation of a parameter (excitation pulses). If the excitation amplitude and/or temporal width overcomes a critical value, the response of the system becomes independent of the excitation itself.

We fix the pump current close enough to the threshold such that intensity is constant in time. We add to the pumping current pulses whose amplitude and width can be varied. Changing the amplitude we check the existence of a threshold [Figs. 8(a) and 8(b)], for the appearance of intensity pulsations whose amplitude and width are independent of the perturbation. Increasing the amplitude of the external perturbation [Fig. 8(c)], the ratio between the number of dropouts and excitation pulses increases. These measurements confirm the excitable character of the system [13].

IV. CONCLUSIONS

The pulse observed under the excitability condition is similar in amplitude and width to those observed in the LFF regime. This provides further evidence for the correctness of our ''dynamical'' interpretation.

It is important to note that this is experimental evidence of an excitable optical system. Theoretically, excitability has been predicted in laser with injected signal [12], but no experimental confirmation was previously obtained.

The theoretical model for semiconductor lasers with optical feedback generally used in previous works (rate equations with a delayed additive term in the field equation) has the possibility of an Andronov bifurcation. In fact, a series of fixed points and saddles are steady-state solutions. However, their relative distance in phase space remains unaltered by changing the pumping current. Thus the saddles would never collide with the stable fixed point with an increase in the current and the Andronov bifurcation cannot take place. But the model uses several approximations which are not fulfilled by our experimental setup, e.g., the amount of feedback, the type of optical cavity, etc. We believe that the most critical assumption of the model is to consider that the laser is a single longitudinal mode and at the same time to introduce a linewidth enhancement (α) factor much larger than the line broadening. This α factor, playing the role of a detuning between atomic and cavity resonances, forces the system to operate at optical frequencies far away from the maximum gain. At the same time the external cavity begins to introduce resonances at higher gain than the solitary laser solution. Such choice in parameter values is clearly unrealistic.

A comparison between our experimental results and a theoretical laser model would require a better description of the atomic equations than is actually offered by the Lang-Kobayashi equations. Two ingredients are required in order to fit a new model to our experimental results: the possibility of an inverse saddle node and the connection between the manifolds of the points. We hope that numerical simulations performed with the model presented in Ref. [14] would confirm the existence of excitability in a laser with optical feedback. In fact, this type of bifurcation could explain the unusual low frequency of the intensity fluctuations of the laser with optical feedback, the strong sensitivity to noise of the interval between pulses, the large amplitude of the pulses independently of the parameter values, the existence of intermittency, and the low standard deviation in amplitude and width of the pulses.

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